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اللسانيات الرياضية : دراسة شامله

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الملخص :

اللغويات الرياضية هي مجال من مجالات البحث اللغوي الذي يطبق الأساليب والمفاهيم الرياضية على الأنظمة اللغوية، أو على ظاهرة تُلاحظ في اللغات الطبيعية من خلال توفير أدوات دقيقة لتحليل البنى والأنماط والعمليات اللغوية. يركز هذا المجال على النماذج الشكلية الناشئة لوصف الظواهر اللغوية مثل بناء الجملة وعلم الأصوات والدلالات والخطاب والبراغماتية، من خلال استخدام المنطق والجبر ونظرية المجموعات. ومن خلال استخدام النماذج الاحتمالية والأنظمة المنطقية، تُمكّن اللغويات الرياضية أيضًا من تحديد الغموض ونمذجة تباين اللغة والتنبؤ بالظواهر اللغوية. يهدف هذا المجال إلى فهم اللغة بدقة وكمية مما يزيد من طرق جديدة لنمذجة الظواهر اللغوية في أنظمة أكثر منطقية. في نهاية المطاف، تُعدّ اللغويات الرياضية مجالاً ديناميكياً يُواصل تطوير فهم اللغة من خلال التقاء الرياضيات واللغويات. تُقدّم هذه الورقة نظرةً شاملةً على اللغويات الرياضية، من خلال استكشاف تطورها التاريخي وأسسها النظرية ومفاهيمها الأساسية ونماذجها الرئيسية ومجالاتها الرئيسية، كما تُؤكّد على أهمية اللغويات الرياضية في سد الفجوة بين الصياغة المجردة والقضايا اللغوية الفعلية. علاوةً على ذلك، تُسلّط الضوء على استخدام أدوات من نظرية المجموعات والمنطق ونظرية الأتمتة والجبر لصياغة قواعد النحو والدلالات والصرف والخطاب وعلم الأصوات. لذلك، يهدف هذا البحث إلى تقديم إطار عمل موحد لفهم نطاق وتأثير اللغويات الرياضية.

الكلمات المفتاحية: اللغويات الرياضية، نظرية اللغة الرسمية ،حساب لامدا، نظرية المجموعات، نظرية الأتمتة

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Mathematical Linguistics: A Comprehensive Overview

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Abstract

Mathematical linguistics is a field of linguistic inquiry that applies mathematical methods and concepts to linguistic systems, or to a phenomenon that is observed in natural languages by providing precise tools for analyzing linguistic structures, ,patterns and processes . This field is focused on emerging formal models to describe linguistic phenomena such as syntax, phonology, semantics, discourse and pragmatics, through the use of logic, algebra and set theory Through the usage of probabilistic models and logical systems , mathematical linguistics also makes it possible to quantify ambiguity, model language variation, and predict linguistic phenomena. This field is aimed to rigorously and quantitatively understand of language which increases new ways to model linguistic phenomena in more logical systems. Ultimately, mathematical linguistics stands as a dynamic field which continues advancements to understand language through the intersection of mathematics and linguistics .This paper presents a comprehensive overview of mathematical linguistics, by exploring its historical development, theoretical foundations ,core concepts, key models and core areas and also emphasizes the importance of mathematical linguistics in bridging the gap between abstract formalism and actual linguistic issues. Moreover, it underlines the use of tools from set theory, logic, automata theory, and algebra to formalize syntax, semantics, morphology, discourse and phonology. Therefore, the goal of this research is to offer a unified framework for comprehending the scope and impact of mathematical linguistics.

Keywords: Mathematical linguistics, formal language theory, lambda calculus, set theory, automata theory

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1. Introduction

Since language is most complex and fascinating aspects of human cognition, so the scientific study of language is linguistics, which means the study that depends on scientific way of explanation, by noticing the form and structures of statement and describe it.

Over centuries, linguists have attempted to understand language's structure, meaning, and use in a logical way , so mathematical linguistics has been emerged as a branch of theoretical linguistics which uses mathematical concepts and methods to study and analyze the structure, form, and patterns of natural languages

Mathematical Linguistics as an interdisciplinary, is sketching on concepts and techniques from different fields such as mathematics, computer science, and philosophy, in order to gain a deeper understanding of the nature of human language and its underlying principles, (De Santo & Rawski, 2022).

Mathematical Linguistics is appeared as a response to the need for more precise and formal methods for analyzing and studying the structure of natural languages. (Ibid.)Since traditional methods of linguistic analysis, such as those used in descriptive linguistics, were often subjective and lacked the rigor and precision necessary for making scientific and formal statements about the structure of language, mathematical Linguistics provides an objective and systematic way of studying and analyzing language, by using mathematical concepts and methods, allowing for more precise and accurate descriptions of its structure and form, (Halliday, 1978).

Studying linguistic phenomena with mathematics is most simple definition one can give for mathematical linguistics, but mathematics is a large field and not all subfields are represented equally in mathematical linguistics.

This research gives a brief explanation about the usage of formal models (major models) such as Chomsky grammars, automata theory, lambda calculus, and formal logic. Although several specialized studies have tackled specific aspects of mathematical linguistics, there is a notable absence of comprehensive research that consolidates the theoretical and practical dimensions of the field. This has led to disjointed understandings and ambiguous boundaries between what is strictly mathematical and what is purely linguistic.

The problem of this study is the lack of an integrated framework that connects mathematical foundations of linguistic analysis with its real-world applications and theoretical evolution. As a result, numerous challenges emerge:

1. A limited understand of the mathematical models which are used in linguistic analysis.

2. Formal approaches and tools of mathematical linguistics have a lack of clear categorization.

3.Formal linguistic theories with language technologies have rare studies that linked them.

4. A focus of other studies is on syntactic structures, ignoring pragmatics, semantics and discourse and diachronic aspects.

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This research aims to fill these critical gaps by presenting a unified, accessible, and rigorous overview of the theoretical foundations, core concepts and major models of mathematical linguistics.

To address the central problem, this study seeks to answer the following research questions:

1.What is mathematical linguistics? Is it an application of mathematical techniques to language, or is it an integrated hybrid discipline?

2.Where and when did the field originate? Who are foundational figures?

3.What are the major models used in mathematical linguistics?

The importance of the research lies in its introduction to mathematical intellectual and its impact on linguistics concepts, while noting the importance of the mathematical concept in formulating terminology, as well as its contribution to bridge a knowledge gap by providing a structured and comprehensive overview of mathematical linguistics.

2. Language and Mathematics

Mathematical language is considered as a language that describes linguistic data because of its tendency to use fewest possible symbols to convey a huge amount of information, which allows to generate infinite expressions according to a limited number of rules.

An extension of normal language , Mathematical language that is used in mathematics to convey outcomes(theorems, proofs, logical deductions, etc.) with clarity, conciseness and precision.

Early studies were focused on the use of language in mathematics such as those by Halliday (1975) and then followed by Pimm (1987), Morgan (1996) and others, who paid more attention to the linguistic form of the mathematical text, particularly its structure and organization, and they considered mathematical text as discourse. As each text has been influenced by communication so that mathematical text which has different patterns of communication (gestures, pictures, signs) has also been. This influence leads to effect on how mathematical meaning is conveyed (Sfard, 2008).

There is no doubt that the relation between language and mathematics is deep and complex for instance mathematics includes symbols, words, and numbers and language is considered as a cognitive tool for explaining these mathematical patterns. Morgan (2000) pointed out that this relationship is not limited to the use of language in oral communication, but rather goes beyond that to include the construction of mathematical meaning through written language. In the same line DeSanto and Rawski (2022) support this idea by stating that “mathematical reasoning cannot occur without linguistic reasoning.”

Since mathematical text is considered as a social activity that involves using a specific language system (Morgan ,2001) ,so understanding mathematical context is required not only to

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realize relation between mathematical terms and numbers ,but it also needs to detain linguistic relation.

Many researchers have pointed out to the difficulties of mathematical language, especially in relation to the differences between everyday language and mathematical language, since mathematical language is characterized by its structure, precision and abstract nature. While everyday language is depended on context, mathematical language is relied on formulas and symbols to represent abstract concepts with exact meaning and these concepts are inherently abstract, dealing with relations and ideas rather than concrete objects, and the symbols which are used could have multiple meanings depending on the context, (Erlwanger, 1973(

To summarize this, mathematical language is a special system deliberated for clarity and representation of abstract concepts so it lists precision and logical rigor ,while everyday language is adaptable and flexible .

3.Mathematical Register

Mathematical register, as a term was firstly introduced by Halliday(1975)that refers to a specific relation between mathematics and language in terms of how the form of mathematical language is shaped and expressed, specifically to wordings, set of meanings and grammatical patterns associated with mathematical activity .Then Halliday developed theory of register to describe variations in language usage by depending on subject matter, social context and communicative purpose.

In Halliday's framework (1978), a register is defined as a "configuration of meanings that are typically associated with a particular situational context".

Consequently, Mathematical register, contains specific vocabulary, syntactic structures, symbols, and discourse features which characterize communication in mathematical contexts. It comprises not only the technical terms such as vector, denominator, or exponent, as well everyday words that are used in mathematical specialized ways, for instance, product, difference, area and table. Moreover, it is involved distinctive grammatical features, for example frequent use of passive voice ("equation is solved by..."), conditional structures ("If x = 2, then..."), and nominalization (turning verbs into nouns and multiplication instead of multiply).

Halliday(1978) also emphasized that learning mathematics involves acquiring this register: "learning mathematics is learning a second language,". This notion underscores the idea that mathematical understanding cannot be separated from the ability to use and interpret its linguistic forms.

4.Mathematical Discourse

It is necessary to refer to the relation between the concepts of language and mathematics and outcomes which are derived from integrating or separating mathematics and language when we determine the position of language in mathematics. Among these results is the appearance of scientific concepts, such as "language of mathematics," or "language of relation".

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Discussion about mathematics as a mathematical discourse began after increasing discussions about mathematics as a social activity, (Morgan , 2001). These discussions are focused on the role of language in creating knowledge and learning mathematics.

Mathematical discourse has attracted the interest of many researchers, including those who are working in the field of linguistic analysis. Among them is Morgan (1996) who pointed out that mathematical discourse can be analyzed by using systemic functional linguistics which is adopted by Halliday(1985) in which language is used as a whole functional system.

5.Mathematics linguistics

Mathematical linguistics has initially served as a foundation for computational linguistics, though its research is being designed machines to simulate natural language understanding is clearly more applied. Inductive methods have gained in applied computational linguistics, whereas deductive logical methods are still very much in vogue in theoretical mathematical linguistics,(Kornai, 2007).

Ivić & Muriel (2013)have stated that "mathematical linguistics is the scientific process that can be performed linguistically in the field of mathematical problems using mathematical numbers." So it is a specialized field which combines principles of mathematics with linguistic analysis.

Since it is focused on using mathematical methods to study patterns, structure and phenomena of languages, both natural and formal. In this respect, mathematical linguistics is very much like the empirical sciences.

Mathematical linguistics is a linguistic construct, based on mathematical and logical formulas, therefore , it is worth noting that “algebraic linguistics” seeks to make “natural language” the subject of study, and “mathematical language” a method for analyzing this language and mathematically re-reducing its parts to achieve “pictorial models” that can be applied computationally, (Kracht, 2003).

Meanwhile Mathematical linguistics is the application of mathematics to solve problems and model phenomena in general linguistics and theoretical linguistics ,wherefore it is assumed to be a non-calculative discipline in the sense that calculations of proportion play no role in its methodology and it is mathematical in that it has been used of sub-disciplines of mathematics, particularly, formal logic and set theory. Gladkij &Mel'cuk (1969) explained that a mathematician is someone who resolves to construct a model of human linguistic behavior. This remark leads to create, a plane of content - a set of meanings from one hand and a plane of expression a set of texts, or utterances ,on the other.

From what has been explained before mathematical linguistics could be defined as the study of the mathematical formal properties of abstract linguistic theories, according to mathematical concepts that represent the linguistic reality which described by them.

6. Historical background

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The development of mathematical linguistics is represented an essential shift in linguistic theory, where language is no longer viewed as a cultural or a social phenomenon but as a formal system acquiescent to rigorous, mathematical treatment. Its origin can be traced to the mid of 20th century, a period of explosive growth in logic, mathematics, theoretical linguistics and computer science. This interdisciplinary convergence set the basis for what is now identified as mathematical linguistics.

To fully understand the origins of mathematical linguistics, one must explore the intellectual contributions of key figures, like Noam Chomsky, Zellig Harris, Richard Montague, and early logicians like Gottlob Frege, Bertrand Russell, and Alfred Tarski.

First of all , Gottlob Frege , a German logician, philosopher and mathematician, is recognized as one of the earliest who apply formal logic to language analysis. His 1879 work introduced a formal language that could express reasoning and logical relations with precision, laying the foundation for formal semantics. Frege's idea of compositionality in which the meanings of parts and syntactic arrangement of a sentence are determined its meaning is still an important principle in modern formal semantics.

Following Frege, Alfred North Whitehead and Bertrand Russell advanced formal logical analysis with their vast work “Principia Mathematica” (1910–1913). Their contributions were pivotal in defining symbolic logic and logical syntax, tools that later demonstrate indispensable in formal linguistic modeling.

One more significant influencer was Alfred Tarski, a logician who developed the concept of model-theoretic semantics in the 1930s. Specifically, his seminal work on the topic was published in 1933, with a key paper being “The Concept of Truth in Formalized Languages”. Tarski's formal theory of truth and his semantic conception of logic have provided mathematical rigor which needed to bridge logic and language in a precise way.

While these theorists were predominantly mathematicians and philosophers, their ideas are extremely influenced linguistic formalism which developed decades later. However, they did not directly create a formal theory of natural language; somewhat, they provided foundational methods and conceptual frameworks which made such a theory possible.

The field of linguistics in the early of 20th century was dominated by structuralist paradigms, particularly in Europe and the United States. The American school of structuralism, where led by figures such as Leonard Bloomfield and later Zellig Harris, emphasized systematic and a scientific approach to the study of language.

Zellig Harris , a linguist who was a pioneer in formal methods for analyzing language structure. Harris sought to describe language in distributional and formal terms. He has presented his approach in linguistics, in his book “Methods in Structural Linguistics”, published in 1951. His approach indicated classifying and segmenting linguistic data using logical and mathematical techniques. Harris's goal was to create a scientific basis for linguistic analysis that was both formal and empirical.

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Under Harris's influence, Chomsky pursued the formalization of linguistic rules and began investigating the mathematical properties of grammars.

Noam Chomsky is viewed as the founder of modern mathematical linguistics. His landmark 1956 paper "Three Models for the Description of Language", introduced formal grammar systems as tools for modeling natural language syntax. In this work and later in Syntactic Structures (1957), Chomsky developed the Chomsky Hierarchy (which will be explained later), which is a classification of formal grammars based on their generative power.

Chomsky's insight was revolutionary because he has showed that natural languages can model by using formal systems like those used in mathematics. This formalization enabled linguistics to treat as a mathematical discipline, subject to theoretical constraints ,proofs and algorithmic analysis. This marked a significant shift from earlier structuralist approaches, which were largely descriptive.

While Chomsky's syntactic models controlled the American scene, European scholars were exploring logic and semantics in relation to language. One of the most influential was Richard Montague.

Montague's work in the late 1960s and early 1970s had culminated in what is now known as Montague's Grammar, a system which is treated natural language syntax and semantics by using formal logic particularly, predicate logic and lambda calculus.

Montague(1970) stated that "There is no important theoretical difference between natural languages and the artificial languages of logicians." This view had challenged Chomsky's assertion that natural languages required a different analytical approach and instead he proposed a unified mathematical treatment of language.

Montague's Grammar (1970) had a profound impact on the development of formal semantics, establishing rigorous mathematical tools to analyze meaning in natural languages. It is influenced on later generations of linguists and philosophers .

From 1970s till now , mathematical linguistics became as a global interdisciplinary field.

7. Theoretical Foundation of Mathematical Linguistics

7.1 Mathematical logic

Mathematical linguistics utilizes mathematical logic to analyze the structure and meaning of language. It employs formal languages and logical systems to model linguistic phenomena, including syntax, semantics, and pragmatics. This approach is allowed for a precise and rigorous study of language, aiding in the understanding of both natural and formal languages. Logical systems provide the framework for reasoning about the formal representations, Ellerman (2017). There are different formal systems used in logic which are explained underneath:

1. Propositional Logic: is used for simple sentence-level semantics and inference rules.

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2. First-Order Logic (FOL): is used in formal semantics to capture quantification, predicates, and relations between entities.

3. Lambda Calculus: is a computational formalism used in Montague Grammar to represent functional semantics, where meaning is computed compositionally.

4. Modal and Temporal Logic: is employed in analyzing modality, tense, and aspect in natural language.

Tarski's (1935) model-theoretic semantics and Montague's (1973) formal semantics are provided perilous bridges between mathematical logic and linguistic meaning. These logical models are valuable in representing meaning, truth conditions, and inference mechanisms, that is bridging the gap between semantic interpretation and syntactic structure.

7.2 Formal Language Theory

A formal language is defined as a set of strings constructed from a finite set of symbols (called an alphabet) , following specific syntactic rules , so it is considered as the heart of mathematical linguistics. This theory is used formal languages and grammars to analyze natural languages .It often involves generative models that describe rules which is governing language structure. Therefore, it provides the formalism that needed to represent linguistic data as a structured system, (Reghizzi, 2009).

An alphabet (usually denoted by Σ) is a finite set of symbols, and a string is a finite sequence of symbols from that alphabet,(Kleene, 1964). Linguistic expressions(sentences or words) can be signified as strings, and entire languages can be realized as sets of such strings. The formedness of strings in a language is determined by grammatical rules. These rules can be distinct by using formal systems such as production rules or regular expressions in a grammar.

Thus, formal language theory provides the first mathematical abstraction of language and treats it as a set of well-formed expressions which can be rigorously manipulated and defined .

7.3 Algebraic Structure

An algebraic structure in mathematics is a set with one or more operations that content exact axioms and these structures form the foundation for abstract algebra, (Finston, & Morandi,2014).

In mathematical linguistics, algebraic structures are used to analyze and model the formal properties of language. They are provided a basis for understanding how linguistic elements combine and interact, drawing on concepts from abstract algebra like groups, semi-groups and rings. These structures help linguists explore the rules and constraints that govern language, in addition to the underlying principles that organize linguistic diversity,(Harris, 1970).

7.4 Automata Theory

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This theory is closely tied to grammar formalisms , which deals with computational machines that recognize formal languages. It is served as mathematical models of computation, (K.Chandusha,2024).

In mathematical linguistics , automata theory is used to represent how syntactic rules could be recognized or processed. Thus, it deals and provides an insight into how linguistic structures can parse and interpret algorithmically. There are four types of automata .As illustrated underneath:

1. Finite Automata (FA)

This automata is used to model regular languages. It is used in lexical spell checkers, analyzers and text search tools.

2.Pushdown Automata (PDA)

This automata can be recognized context-free languages and handle recursive structures, which makes it suitable for syntactic parsing in natural language processing.

3.Linear Bounded Automata (LBA(

It is used to recognize context-sensitive languages. This automata is capable of capturing more complex linguistic constraints.

4.Turing Machines

Turing machines represent the most powerful computational model and are capable of recognizing recursively enumerable languages, (K.Chandusha,2024).

Automata theory thus provides the computational backbone of mathematical linguistics, allowing researchers to study the decidability, recognizability, and complexity of various linguistic structures.

8. Core Concepts of Mathematical Linguistics

Concepts are linguistic ideas expressed in mathematical terms, not the actual devices or subfields .These specific abstract ideas are used to describe how language is reasoned and viewed mathematically. These concepts are illustrated in the following:

1. Language as a Formal Object

A natural language is viewed as a set of strings over an alphabet and a system of rules and symbols , similar to logical or mathematical systems, rather than focusing on its communicative or social aspects. In this perspective , language could be analyzed depending on its structure and how it's formed, rather than its usage in context or meaning, (Reghizzi, 2009).

2.Grammar

In mathematical linguistics, grammar is referred to a set of rules which is defined how symbols could be joined to form right strings, that could be analyzed by using mathematical models like formal grammars and automata, that help in understanding the underlying principles of language structure. Basically, grammar here is provided a formal framework for analyzing and describing languages, both formal and natural , by using mathematical concepts,(Maienborn, et al. ,2011)

3.Generativity

In spite of languages have a limited number of sounds, words, and grammatical rules, that could be combined in countless ways to form understandable and new sentences. This is referred to "generativity" that is used to describe linguistic theories or models which are based on the idea that a single set of rules can explain how all the possible sentences of a language are formed (Van Benthem and ter Meulen 2010).

4. Compositionality

In mathematical linguistics, compositionality is referred to the principle that the meaning of a complex expression is determined by the meanings of its simple parts and the rules that are used to combine them. This means that understanding a complex sentence needs to combine the meanings of its individual words (or morphemes) according to the grammar rules. This principle is fundamental to formal semantics , allowing for the analysis of possibly infinite sentences from a finite set of rules and meanings,(Baggio, et al. ,2012)

5.Derivation

In mathematical linguistics, derivation is devoted to the process of transforming a linguistic structure into another by using a set of rules, (Chomsky,1957). It is considered as a core concept in formal language theory and generative grammar, where the aim is to determine a formal system which could be generated all grammatical sentences of a language. This is involved mapping linguistic expressions (such as strings of words) to their parallel structural descriptions (such as parse trees) relied on grammatical rules,(Eisenbud,1999).

6.Ambiguity

Ambiguity in mathematical linguistics is denoted to the potential for a mathematical expression or statement to have multiple valid interpretations, which leads to confusion about its intended meaning. This could be raised from various factors, such as the usage of imprecise language, notation that is open to multiple interpretations or omission of important contextual information, (Abramovits & Stegun, 1964)

7.Learnability

In mathematical linguistics, this term is referred to the study of how a finite set of data such as a limited set of sentences could be used to learn the complete structure of a language, (Kracht, 2003) . Wherefore, it referred to the study of how a language, either natural or formal, can be acquired by human through exposure to data. Mathematically it is analyzed how language acquisition can be occurred in principle, exploring the feasibility of a learning algorithm to discover aspects of syntax or other linguistic features. (Ibid.)

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8. Entropy

In mathematical linguistics, entropy is quantified the average of uncertainty or content of information that is associated with a language,(Shannon&Weaver,1949).It is measured how unpredictable the next element (e.g., letter, word) is given the preceding context. Higher entropy is indicated complexity and unpredictability , whereas lower entropy is referred to more redundancy and predictability ,(Pathria,&Beale,2011).

7. Major Models of Mathematical Linguistics

Mathematical linguistics is employed various models to analyze and represent language. Major models include formal grammar theories (like Chomsky's hierarchy) for syntax, logical and set theory approaches for semantics. Graph theory, and probabilistic models for pragmatics as explained below:

1. Chomsky's Hierarchy

This hierarchy is classified formal grammars based on their generative capacity. It is involved the following types:

1. Regular Grammars

This type is distinct by rules that generate strings without recursion. it generate languages, that can be recognized by finite automata. This type is used in pattern matching and lexical analysis, (Chomsky, 1956).

2. Context-Free Grammars (CFGs)

This type is generated context-free languages, that are recognized by pushdown automata. These are widely used in syntactic analysis such as subordinate clauses and parentheses, (Chomsky, 1956).

3. Context-Sensitive Grammars

This type is used to generate languages recognized by linear bounded automata. These grammars are allowed rules that can depend on the surrounding context of symbols .Some phenomena in natural language, such as agreement and cross-serial dependencies, are modeled with it, (Chomsky, 1956).

4. Recursively Enumerable Grammars

This type is generated the most general class of languages, corresponding to Turing machines. They can define any computable language but are not guaranteed to be decidable.

Chomsky's hierarchy is not only offered a taxonomy of language complexity but also informed parsing strategies and language recognition algorithms, (Chomsky, 1956).

2. Logical Approaches

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2.1 Logic and Semantics:

Mathematical linguistics is used logic (propositional logic, predicate logic, modal logic) to formalize the meaning of linguistic expressions and to reason about semantic properties like entailment and presupposition, (Kracht, 2003).

2.2 Game-Theoretic Semantics:

This approach is analyzed the meaning of language and logical formulas in terms of games in terms of games by mounting meaning-constituting situations which are played between strategic players, such as a language user (speaker) and a listener, (Ibid.).

2.3 Non-monotonic Logics:

These logics is dealt with situations where new information can invalidate previously drawn conclusions, relevant for modeling aspects of natural language like default reasoning,(Ellerman ,2017).

3. Graph Theory:

In mathematical linguistics, graph theory uses structures of vertices (nodes) and edges (links) to model relationships and hierarchical structures within language. Consequently , this theory is helped represent sentence syntax (dependency graphs) ,analyze complex grammatical structures and model relations between words, (Ibid.).

4. Probabilistic Models

In mathematical linguistics, probabilistic models are more than just "engineering tricks" for natural language processing. Additionally, they have theoretical uses. These models are included the following types :

4.1 Bayesian Models:

This type of probabilistic models is used Bayesian methods to analyze data and make predictions in areas like pragmatics, sociolinguistics, and historical linguistics,(Hacking, 1967).

4.2 N-Gram Models:

These models are investigated as stochastic processes on symbolic sequences in mathematical linguistics. They are used to examine a language's statistical structure, redundancy, and entropy, (Ibid.).

4.3 Markov Hidden Models:

HMMs are abstract probabilistic automata that are examined as formal tools in mathematical linguistics to simulate probabilistic connections between hidden language categories such as phonemes or syntactic classes, (Ibid.).

5.Set Theory

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This theory is essential to mathematical linguistics because it offers a framework for describing and evaluating linguistic processes. In order to formalize linguistic notions and advance theories about language, it provides a means of defining and working with groups of linguistic objects, such as words, phrases, or grammatical rules,(Ibid.). Applied set theory can be seen in the allophonic variants of each phoneme in a language, as well as in semantic classes, word classes, and natural classes.

8 . Core Areas of Study in Mathematical Linguistics

Since mathematical linguistics is the theoretical and formal study of language that used tools of mathematics, its core areas ,which are categories or branches of study ,include syntax, semantics, phonology, morphology and Discourse. Each area is applied specific mathematical concepts such as formal grammars, automata, logic, and algebra to define and analyze linguistic structures rigorously, as explained in the following:

8.1 Syntax

Syntax is concerned with the structure of sentences. In mathematical linguistics, syntax is described as being used formal grammars which define how phrases and sentences are formed from words combination. Wherefore, mathematical linguistics uses formal grammars (such as context-free grammars (CFGs), regular grammars, and tree-adjoining grammars(TAGs) to model syntactic structures. CFGs can accurately describe nested syntactic structures,(Chomsky,1956).

Since the central goals of mathematical linguistics is to understand the generative capacity of different types of grammars, which types of sentence structures people can or cannot produce, so syntactic trees , parse structures and derivations are treated as mathematical objects, which can be analyzed, manipulated, and generated and this is allowed for syntactic validation.

Mathematical linguistics is used formal syntactic rules to analyzed sentence structure efficiently and automata theory to represent how those syntactic rules can be recognized or processed. Thus, it provides an insight into how linguistic structures can be interpreted and parsed.

Modal logic is used to model syntax by employing different grammatical moods. Since syntax is provided linguistic intuition, wherefore, mathematical linguistics in this respect is supplied formal tools and frameworks to model and analyze those structures, (Kracht, 2003).

8.2 Semantics

The study of meaning in the language is Semantics .It deals with how meaning is expressed by words, phrases, and sentences, as well as how that meaning can be formally represented and interpreted, (Heim& Kratzer,1998). In mathematical linguistics , the formal system is used to represent and comprehend meaning through the usage of tools like predicate logic and lambda calculus to model the relations between linguistic expressions and their meanings and analyze how the meanings of individual words combine to form larger meanings . Semantics can be unified under this system by relying on Richard Montague's framework from the 1970s, which permits the conversion of sentences into logical expressions while reserve their meaning.

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8.3 Phonology and Morphology

Phonology focuses on sound systems of languages. Whereas, morphology is studies word formation. In mathematical linguistics, phonology and morphology are studied by using formal ,mathematical, and computational methods, such as (CFGs) to model linguistic structures, (Kaplan & Kay, 1994), thus mathematical linguistics , in this respect ,is provided a framework for analyzing how these complex systems could be represented mathematically by using formal rules and hierarchical structures which is governed the interaction between word forms and sounds.

8.4 Pragmatics and Discourse

In mathematical linguistics, Pragmatics is studied how the use of language, mainly its context-dependent and intended meanings (away from literal semantics), and its influence on the understanding of mathematical concepts and texts, while discourse analysis is examined the content and structure of mathematical texts to understand how meaning is built and how mathematical understanding co-created, (Austin & Howson,1979).

Both fields are highly interrelated, with pragmatics is provided tools to understand "why" and "how" of language usage within the textual structures analyzed by discourse analysis, principally in mathematical discourse. Pragmatics and discourse are used probabilistic models and game theory to analyze language usage in context, speaker intention, and conversational structure, (Ibid.) .

9.Conclusion

The origin of mathematical linguistics cannot be attributed to a single moment, place or person. Rather, it is the result of collaborative interdisciplinary efforts that are illustrated from philosophy, logic, mathematics, linguistics, and computer science. Key figures like Frege, Tarski, Harris, Chomsky, and Montague contributed the necessary intellectual frameworks, while institutions like MIT, Stanford, and European universities are provided productive ground for this field's development.

Mathematical linguistics formally emerged in the 1950s but stands on giants works from the late 19th century. It is a discipline appeared in order to understand human language with the precision of mathematics, and it is a goal that is continued to drive research across linguistics and cognitive science.

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